Mathematics Standards

Who Needs Them?

Using the current standards, . . . teachers can create learning environments that actively engage students in stimulating hands-on, minds-on learning.

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The National Council of Teachers of Mathematics (NCTM) has been at the forefront of creating mathematics standards for more than 20 years. Using the current standards, *Principles and Standards for School Mathematics*,[1] teachers can create learning environments that actively engage students in stimulating hands-on, minds-on learning. In the two vignettes that follow, we will describe non-standards-based classrooms, followed by an outline of the new standards, some international perspectives, and the five important process standards. Finally, we will revisit the two classrooms as they are redesigned to apply a standards-based approach to teaching and learning.

Ms. Allyn's Classroom: A Vignette

First, let's visit Sylvia Allyn's 3rd-grade classroom, where she is introducing the basic facts of addition. Ms. Allyn emphasizes to students the importance of memorizing the 100 addition facts, as they will use these facts for the rest of their lives. After distributing the addition facts table, she explains how to use it (see Figure 1), saying that when adding 3 and 5, you place a finger on the row starting with a 3 and a finger from the other hand on the column starting with a 5. The intersection of where the two fingers are pointing is the correct answer, namely 8.

After giving many examples of how to use the table, Ms. Allyn turns her attention to assessing whether students can use the tables to obtain a desired answer. When she feels satisfied with their proficiency, she tells them to practice each basic fact until they can respond immediately to an assigned problem. Ms. Allyn then methodically works her way through the different fact families, starting with the 0 facts, then the 1 facts, and continuing through the 9 facts. She uses flash cards and speed tests to assess her students' progress. After several weeks, the grade book shows that

Figure 1
Addition Basic Fact Table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

most students have successfully passed their fact family tests.

But do they really understand what they learned? Let's ask each child several questions from the fact families. As we randomly jump from one fact family to another, most students have difficulty responding. They revert to counting on their fingers or give incorrect answers.

Mr. Smith's Classroom: A Vignette

Moving down a few doors, we come to Robert Smith's 6th-grade classroom. Mr. Smith has just finished reviewing the area formulas for squares ($A = s^2$), rectangles ($A = lw$), triangles ($A = \frac{1}{2} bh$), parallelograms ($A = lw$), and trapezoids [$A = \frac{1}{2} h (b_1 + b_2)$]. To test students' knowledge of these formulas, he asks them to find the areas of the following figures:

1. A triangle when $b = 2$ cm and $h = 4$ cm.
2. A parallelogram when $l = 3$ cm and $w = 4$ cm.
3. A square when $s = 3$ cm.
4. A trapezoid when $h = 2$ cm, $b_1 = 3$ cm, and $b_2 = 4$ cm.
5. A rectangle when $l = 2$ cm and $w = 3$ cm.

All students score highly on the quiz, and Mr. Smith feels satisfied that they understand the concept of area.

To check his perceptions, let's give the students a quiz, using a triangle and the letters l and w for the base and height, respectively. Many students focus on the letters and use the rectangle or parallelogram formula ($A = lw$) to calculate the area. When we probe their understanding by asking for an explanation of their answers, most of their students focus on the mechanics

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of inserting numbers in the formulas, deciding which operations to use, and calculating the answers. Further prodding shows that few students can relate their answers to the figures. For example, they cannot describe geometrically how 6 cm^2 relates to the covering of a rectangle 2 cm long and 3 cm wide.

Can the new mathematics standards help solve these problems?

A Look at the New Standards

The new *Principles and Standards for School Mathematics*, found on the World Wide Web at http://standards.nctm.org, calls for more active student involvement in mathematics learning, writing and speaking about mathematics, negotiating for consensus in small groups, and open-ended problem-focused teaching. "Knowing" and "doing" mathematics is no longer defined as memorizing number facts and applying computational procedures and algorithms. Instead, the standards emphasize conceptual understanding and the ability to reason and communicate with others to effectively solve problems. Specifically, *Principles and Standards for School Mathematics* "is intended to

- set forth a comprehensive and coherent set of goals for mathematics

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for all students from pre-kindergarten through grade 12 that will orient curricular, teaching, and assessment efforts during the next decades;

• serve as a resource for teachers, education leaders, and policymakers to use in examining and improving the quality of mathematics instructional programs;

• guide the development of curriculum frameworks, assessments, and instructional materials;

• stimulate ideas and ongoing conversations at the national, provincial or state, and local levels about how best to help students gain a deep understanding of important mathematics."[2]

An International Perspective

From an international perspective, students in the United States frequently are not competitive with students from other countries. The United States ranked 28th among 41 countries in a 1997 U.S. Department of Education study. The Third International Mathematics and Science Study (TIMSS) compared the United States to third-ranked Japan. Students in the United States spent much less time on thinking activities than their counterparts in Japan (24 percent as compared with 71 percent). Conversely, United States students spent much more time on skill activities than Japanese students (60 percent compared to 24 percent).[3]

Another stark contrast is the different tasks in which students engaged while doing seat work. Japanese students spent 41 percent of their seat work time on practicing procedures, 15 percent on applying concepts, and 44 percent on inventing and thinking activities, as opposed to American students, who spent 96 percent of their seat work time on practicing procedures and virtually no time applying concepts or doing activities related to inventing and thinking.[4]

Summarizing TIMSS, the U.S. National Research Center in 1996 made several major findings, the first stating that the United States "standards are unfocused and aimed at the lowest common denominator. In other words, they are a mile wide and an inch deep."

NCTM Process Standards

For the United States and other countries to improve their international rankings, teachers must emphasize five important process standards: (1) Problem-Solving, (2) Reasoning and Proof, (3) Communication, (4) Connections, and (5) Representation.

Problem-Solving, or engaging in a task when one does not know the solution in advance, is integral to all standards-based mathematics learning. In order to find solutions, students must draw upon their mathematical knowledge and, by doing so, construct new mathematical understandings and develop new problem-solving methods.

By involving students in mathematical *Reasoning and Proof*, teachers help them learn to think analytically and to perceive patterns and structure in real-world situations and mathematical abstractions. Often, the only practice students get in working with mathematical proofs is in their high school geometry classes.[5] However, this process standard seeks to involve all K-12 students. Younger children will rely more on physical objects to reason and justify their answers, often employing trial-and-error methods. As they progress to the upper elementary level, the teacher should guide them in using more systematic arguments to justify their answers.

The *Communication* standard encourages students to reflect, refine, discuss, and amend their thinking. Students are more likely to achieve greater mathematical understanding when challenged to communicate the results of their thinking orally and in writing.

The *Connections* standard emphasizes the need for students to study and learn about the entire field of math, both within a particular grade level and among various grade levels. Students who make meaningful mathematical connections learn to recognize and apply mathematics in various contexts.

Various forms of *Representation*—such as diagrams, graphs, equations, and other symbolic expressions—are essential foundations for students' conjectures and help enrich their understanding. Math classes should involve students in selecting, applying, and translating among various representations to solve problems.[6]

With these five process standards in mind, let's revisit Ms. Allyn and Mr. Smith's classrooms as they begin to implement a more standards-based approach to teaching and learning.

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Ms. Allyn's Standards-Based Classroom

Instead of focusing solely on drill, Ms. Allyn now helps students understand how to use a variety of strategies to learn facts efficiently. For example, over the course of the unit, she helps them make connections between their knowledge of the standard number word sequence (1, 2, 3, . . .) and the *one-more-than* and *two-more-than* facts. Since Ms. Allyn's students already know that 6 follows 5, she helps them connect this to the fact that both 5 + 1 and 1 + 5 equal 6. Rather than having students learn 7 + 2 = 9 in isolation, she has them focus on saying 7 and then counting two more ("7, 8, 9"). As she watches her students work in groups, she notices that discussion and verifying the facts of others are key components in aiding their understanding.

Ten-pairs are easily taught by combining Unifix Cubes of two different colors to make a total of 10 cubes. Ms. Allyn models several examples such as 1 blue and 9 red, or 2 blue and 8 red and then gives students opportunity to form 10-pairs on their own. Creating models helps students learn these facts.

Ms. Allyn spends a lot of time on *double facts* because she realizes that many of the basic facts center around knowing these. She uses visual representations to form the doubles. For example, she holds up a picture of a bug to illustrate "Bug Double" or 3 + 3 = 6. Similarly, she has Spider Double and Hand Double represent 4 + 4 = 8

Standards help promote quality when they become a benchmark for preparing, implementing, and reviewing instructional practices and learning opportunities.

and 5 + 5 = 10, respectively.[7]

A related set of facts is the *Near-Doubles facts.* As the name suggests, these take the form of $x + (x + 1)$ or $(x + 1) + x$. For example, 6 + 7 and 4 + 3 are both near-double facts. Ms. Allyn shows students how to reason through the problem, 6 + 7, using the double facts. She explains that 6 + 7 is like the double fact 6 + 6, except you add 1 more. Knowing that 6 + 6 = 12, students quickly add 1 to 12 and obtain 13. Students begin to build connections as they see relationships between related facts.

Ms. Allyn's newly designed classroom helps students make connections between ideas, use hands-on methods of teaching addition facts, draw on their reasoning skills in justifying their answers, and encourages them to communicate with one another about math. As a result, their knowledge is much richer and more interconnected, and they are better able to remember and apply what they have learned.

Mr. Smith's Standards-Based Classroom

Mr. Smith no longer emphasizes the memorized formula from the textbook. Rather, he emphasizes the relationships between the areas of the different figures. He ties the concept of "area" to the covering of the geometrical figures. For example, he gives each student a paper with the following figures on it: a square 2 cm by 2 cm, a rectangle 2 cm by 3 cm, a parallelogram 2 cm by 3 cm, and a triangle with base 2 cm and height 3 cm. He asks the students to work in small groups to figure out how to cover each figure with 1 cm by 1 cm squares (see Figure 2) and to determine the area of each figure. As he looks over the shoulders of his students, he sees them partitioning the figures in different ways. They divide the square into four smaller squares and add them together, and use the same approach with the rectangle. Some of them have trouble with the parallelogram. A few partition parallelogram KLMN using a dashed line segment NL (see Figure 3). They then slide triangle KLN to the right to form rectangle LSMN. Next, they use their knowledge about the areas of rectangles to find the area of the original parallelogram. As they do this, the students begin to see why the formulas for parallelograms and rectangles are the same. One group of students works on triangle OPQ by adding another triangle of equal size, making it parallelogram OPQR. Once they calculate the area of the parallelogram, they take half to determine the size of the original triangle. Students begin to see that the area of a triangle is one-half that of a parallelogram. After they spend sufficient time on the problems, Mr. Smith asks several groups to describe their problem-solving processes for the entire class.

Figure 2
Mr. Smith's Chalkboard Problems

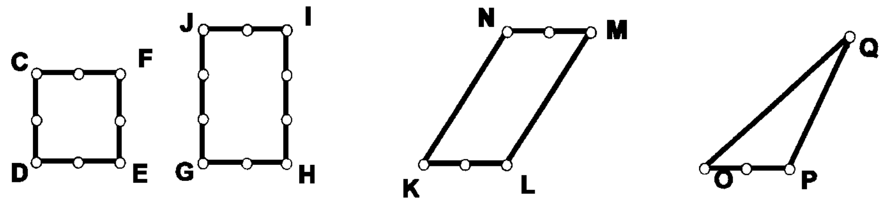
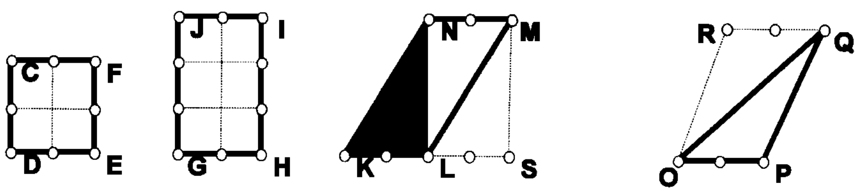


Figure 3
Mr. Smith's Students' Solutions



Over the next several days and after using many examples, Mr. Smith has his students create formulas for the areas of the squares, rectangles, parallelograms, and triangles. Students notice the areas of the square, rectangle, and parallelogram all are $A = lw$. They refer to each figure using the words *length* and *width* and observe that the area of the triangles is half that of a parallelogram, $A = \frac{1}{2} lw$. Mr. Smith is impressed at the connections and relationships his students have been able to make when allowed to conjecture and explore.

Next, Mr. Smith presents the trapezoid and asks his students how to determine its area. Within a couple of days, several groups of students have come up with different ways to do so. The first group duplicates the original trapezoid ABCD and attaches it to the trapezoid along side CD, making it a parallelogram, ABEF. They determine its area using $A = lw$, or (12 cm + 6 cm) times 2 cm = 36 cm^2, then take half of the area (18 cm^2), to determine the area of trapezoid ABCD (see Figure 4 on page 25).

Students in the second group cut trapezoid ABCD through its middle and rotate the top portion (shaded region) around Point F to make parallelogram AEGH. The height of this parallelogram is half that of the original trapezoid, and its length is the sum of the bases of the trapezoid (6 cm and 12 cm). So the area of the trapezoid ABCD is equal to the area of parallelogram AEGH, which equals ½ (2 cm)

Figure 4
First Group's Trapezoid Problem

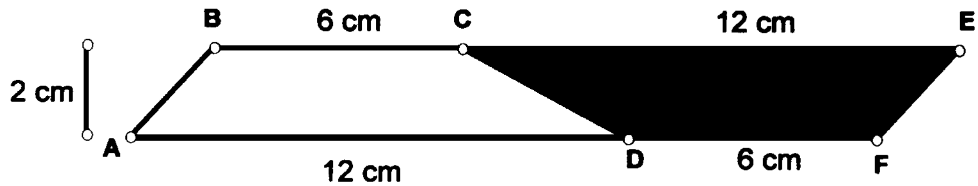
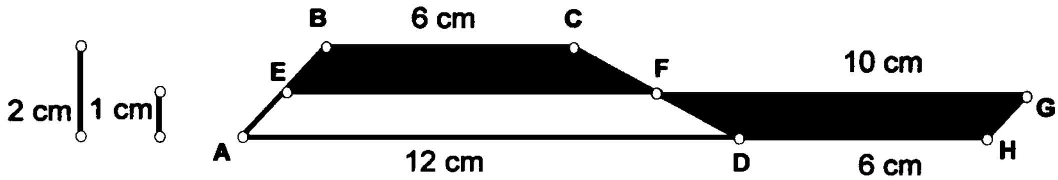


Figure 5
Second Group's Trapezoid Problem



(6 cm + 12 cm) = 18 cm^2 (see Figure 5).

In solving these problems, students are able to visualize the mathematical operations as well as describe in geometric terms the traditional trapezoid formula, $\frac{1}{2}h\,(b_1 + b_2)$. They make connections between the areas of the trapezoid and the parallelogram because they have time to explore, debate, challenge, and discover. The activities help them make sense of mathematics.

Conclusions

Teachers need standards to ensure quality, to set goals, and to promote necessary change. Standards help promote quality when they become a benchmark for preparing, implementing, and reviewing instructional practices and learning opportunities. Although they are not prescriptive, they do set the stage for meaningful decisions about teaching and learning. Standards help teachers to reflect upon and question their practices and then make appropriate modifications to improve teaching and learning.

The authors believe that *Principles and Standards for School Mathematics* promotes a vision for the mathematics classroom that

- Fosters active involvement in learning;
- Views teachers and students as thinkers, doers, investigators, and problem solvers;
- Asserts that all students can learn;
- Encourages teachers to reflect on their teaching;
- Uses real-life situations to make connections to mathematics and science; and
- Calls on parents, teachers, principals, and superintendents to be partners in change.

We must challenge ourselves and our students through a standards-based teaching and learning environment.

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