What Do Debt, AIDS, a

HAVE IN COMMON?

Recently, while we were visiting a cousin near San Diego, California, seven junior high school students dropped by to visit her. I asked them, "What's your favorite subject?"

"Math!" almost all of them answered without hesitation.

I thought ruefully that most of these kids will hate math by the time they get to college. Why? One of the reasons I heard as a college professor was that students figured they would never use mathematical equations, terminology, or geometric proofs in their daily lives.

Yet throughout life, we and our students will have a variety of problems that can be solved using mathematics, which provides tools to successfully deal with personal finance, to analyze risk, and to interpret statistics.

In this article, we first examine several financial topics that play an important part in everyone's life. We also look at an example in statistics concerning the ever-present threat of HIV/AIDS. To conclude the article, we present a fun example that can be stretched into some interesting mathematics and possibly even an opportunity to earn money or start a career. Some of these examples have been drawn from the course Applications of Mathematics that I have taught at La Sierra University in Riverside, California.[1]

Examples like these, modified appropriately to the mathematical level of the students, can be used in middle school and high school classes as well as with college students.

Financial Literacy

Adventists have long regarded faithful stewardship as a vital part of the Christian life. In fact, Stewardship is doctrine number 21 in our list of Fundamental Beliefs.[2] Stewardship is defined as using our resources of time and possessions in a wise way for our own support and having enough to bless others and enable the ministry of the gospel and growth of the church.

Generous stewardship is commended in the Bible, which promises prosperity to those who are generous in tithes and offerings. For example, in Malachi 3:10, God promises that if we return a full tithe, He will give us so great a blessing we won't have enough room to store it all. Illustrations of how this promise has been fulfilled include 1 Chronicles 29, where David praises God that the people are so wealthy that they have been able to give great treasure for the construction of the temple. On the other hand, in Haggai 1, God withholds dew and rain because His people neglected to provide for the Lord's house.

Every student will need to know how to manage money, both now and in the future. Debt can pose a major challenge for many people and can seriously threaten their lifestyle and future earnings. Many young people in the United States have student loans that they will need to start repaying shortly after graduation and which will threaten their financial solvency for many years until they are paid back. Even declaring bankruptcy does not clear these loans.

For example, CNN reports that the average American college graduate had a debt of $35,200 in 2013.[3] Most graduates from Adventist colleges and universities will also have significant indebtedness. Students who drop out of school without graduating will have an even harder time paying back their loans because they have fewer job opportunities. But whether or not they graduate, students incurring government loans must pay at least $50 per month and repay their loans within 10 years.[4] When this article was written, the lowest interest rate on direct loans to the student was a fixed rate of 3.86 percent.[5] The amount of interest varies, depending on the source of the loan. Many parents are also taking out personal loans or refinancing their homes in order to pay for their children's education, which creates a huge amount of extra indebtedness and serious challenges for those who, for a variety of reasons, become incapable of keeping up with payments.

In the Applications of Mathematics class, we talk about in-

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debtedness and repayment of loans. I supply students with three compound interest formulas that they can use in class and on tests and exams.[6] Using the monthly payment formula shown in Endnote 6 with a *principal* = $40,000, *rate* = 0.0386, and *years* = 10, the graduate will be paying $402.33 per month. This becomes sobering when the student realizes that if he or she is working for a minimum wage of $8.00 per hour, just repaying the loan will require more than 50 hours of work every month. To make matters worse, over those 10 years, the student will pay a total of more than $8,000 just in interest. With a Perkins loan, the interest rate is 5 percent, making the monthly payment $424.27, and the person taking out the loan will have to pay almost $11,000 in total interest.

Besides student loans, the average person will incur debt for automobile loans and home mortgages, assuming he or she buys a car or a house or both. Apartment dwellers will have to pay rent every month. Other major expenses include food and clothing, cell phone service, computer-related fees, and perhaps

the cost of a wedding and children. There will be unavoidable expenses such as insurance, health care, and various types of taxes. Planning and budgeting for these expenses must be part of everyone's education.

Probably the worst kind of debt most people accrue is credit-card debt. A recent monthly credit-card statement from my bank showed a balance of $1,259.24 and asked for a minimum payment of $35.00. Recently, the U.S. Government has required such statements to include an estimate of the length of time it will take the borrower to pay off the debt and the amount he or she will actually pay. This statement revealed that at $35.00 a month, it would take me about four years to pay off this debt, and I would end up paying $1,696.00. If I chose to pay $44.00 a month, this would reduce the repayment time to about three years, and I would have to pay $1,578. In other words, I would be giving the bank between $318.00 and $436.00. As you may have guessed, I wrote the bank a check for $1,259.24 and paid no interest. Using formulas like the one "Monthly Payment on Loan" and the other two formulas in Endnote 6, students can estimate the size of payments they'll be making and how much they can save by paying the whole amount. After a few such calculations, a wise consumer and good steward will quickly decide he or she can more profitably put credit-card interest to much better uses.

One occasionally hears about "good debt" and "bad debt."[7] "Good debt" is defined as debt incurred for expenditures that will pay dividends, enhance one's earning power, or increase in value over time (i.e., buying property or an automobile, or investing in advanced education). It may also include expenditures for preventive maintenance on an automobile or house. "Bad debt" is debt incurred for expenditures that do not convey any long-term benefits. However, care must always be exercised not to take on so much debt, even of the "good" variety, that a person cannot pay it off with regular earnings.

Some Web pages you may wish to include in your repertoire for teaching practical mathematics include the following:

• http://www.mymoney.gov/, a free source sponsored by the U.S. Government that includes a wealth of resources to help people in all of their financial needs.

• http://www.xmarks.com/suggest/topic/financial_planning, which links to a number of sites including the MyMoney site mentioned above.

• http://www.daveramsey.com/. Financial consultant Dave Ramsey provides many resources relating to personal finances such as handling debt, life insurance, and mortgages.[8]

You may also wish to follow some Twitter sites, such as these:

• @TeenFinanceEdu, which promotes teen economic independence.

• @A_Lusardi, a site created by Annamaria Lusardi, an academic and economist who is passionate about financial literacy.

• @ManishaThakor, which contains a wealth of advice on investing.

• @EARN, which provides much advice, especially for those with lower incomes.

• @CreditExperts sponsored by "Credit.com where credit and money experts offer advice and free, easy-to-use tools to help you save money and make smart financial decisions."

Probability

Is it safer to drive or to fly? Should you invest in stocks and bonds or hide the money under your mattress? Human beings encounter events and circumstances that require them to choose between two or more alternatives every day. It's helpful to be able to estimate numerical probabilities or odds for each event. For example, the U.S. National Safety Council gives the odds for death when one travels by car at 1 in 492 (a probability of approximately 0.00203), and when one flies in a commercial airliner, at 1 in 8,357 (0.00012).[9] It is usually very difficult to assign numerical probabilities to everyday events, and not everyone will agree with the level of risk as calculated by someone else.[10] However, some activities are inherently risky, and statistics can alert students to the significant dangers of mimicking their peers who are texting, speeding, or drinking while driving; or who are experimenting with dangerous drugs.

Knowing how to find the probability of various outcomes is another useful tool provided by math. As I write this article,

Quicken Loans has just announced a billion-dollar contest. To win, a contestant must submit a perfect bracket for this year's National Collegiate Athletic Association (NCAA) tournament; in other words, he or she must correctly predict the outcome of all 64 basketball playoff games. Quicken Loans has contracted with Warren Buffet to insure the contest. What are the odds of winning? Assuming that the contestant's guesses are purely random, the probability of predicting the outcome of any single game is exactly 50 percent. So a person's chance of winning is one in 2^{64} or 1 in 18 quintillion (18 followed by 18 zeros). These odds can, of course, be improved significantly by studying the performances of the teams. Warren Buffet's company Berkshire Hathaway has calculated that no one has a better chance of winning than 1 in 4,294,967,296.[11]

The average number of lightning deaths in any one year in the United States is 51.[12] Since there are approximately 320 million people in the U.S., this gives each of them a chance of 51 in 320 million; thus, the chance of dying from being struck by lightning is 1 in 6,274,510. So comparing these two events, any contestant is more than 600 times more likely to die from a lightning strike than he or she is to win the Quicken Loans contest.

Gambling

Typically, gambling is defined as playing a game where one can win or lose money or possessions. The Quicken Loans example above is technically not gambling because no one has to pay to play or loses money playing it. Many people gamble in the hope of getting something for nothing—without earning it. Gambling can be just as habit forming for some people as alcohol or tobacco is for others.[13] Participating in gambling games is frowned on by the Adventist Church, whose official statement of belief about gambling gives several reasons for this stand, including that it "violates Christian principles of stewardship;" causes crime, and creates false hopes.[14]

Students can be shown how to calculate the expected winnings/losses in games of chance like dice, card games, roulette, and the many lotteries promoted by casinos and state governments. The mathematical expectation is the amount that a player can expect to win or lose on each game if he or she continues to play the game. Students can learn to calculate the mathematical expectation as the product of the probability of an outcome times the amount that is won or lost on that outcome. However, the mathematical odds may not be accurate because gambling games at casinos and lotteries are often rigged to favor the owners. Most students will come to see that continued gambling will eventually ruin them.[15] Clearly, for the Christian, gambling is poor stewardship and a bad investment of the money God has entrusted to us.

Risk Taking

There are other forms of risk taking that are regarded as more or less reputable and trustworthy. These include the following:

• *Insurance.* Insurance companies spend a great deal of money calculating various probabilities and then fixing premiums to cover potential losses and to ensure that the company makes money. In actuality, when we insure our property, we are participating in a community endeavor to help people who suffer specific types of misfortune on their insured property. Of course, we are also paying for company profit and for the lawyers insurance companies retain to defend both us and their decisions. When people finance a home or register a motor vehicle, they are required to have insurance.

Many countries in the world have some form of socialized medicine so that their citizens do not have to bear the huge medical expenses that can occur from even a single serious illness or injury. The Federal Affordable Care Act in the United States is designed to ensure that its citizens also have insurance that will help cover the expenses or preventive care and medical emergencies. The Adventist Church encourages its institutions and members to purchase sufficient property and health insurance for their needs.[16]

• *Stocks and Bonds.* Since 2000, the North American Division has enrolled its employees in a defined contribution retirement plan and has encouraged workers to invest in this fund.[17] The church also invests a certain amount in the plan and matches up to a certain percentage of the employee's contribution.

The most popular retirement plan investment is in mutual funds, which are combinations of stocks and bonds. There is no guarantee that these funds will earn a profit, and some may, in fact, decrease substantially in value, as was discovered by employees who invested in the NAD retirement fund between 2000 and 2010. A graph depicting the rise and fall of the U.S. stock market[18] would show that, on the average, during the past hundred years or so, it has performed well. But there have been periods where it has done poorly: 1929 to 1948, 1966 to 1983, and 2001 to 2009. For example, around June 1931, the Dow Jones Industrial Average (DJIA) plummeted from around 5000 to less than 1000; a drop of more than 80 percent, an event that coincided with the beginning of the Great Depression. The third worst performance of the U.S. stock market occurred in 2008, when the DJIA plunged almost 34 percent. In the years since then, the market has recovered rapidly, breaking all records—at least at the time when this article was written. But will it last?

Investing always involves a certain amount of risk. If a couple retired in 1965 with a big stock portfolio, it would likely have depreciated steadily to less than one-third of its original value over the next 18 years; it would not have returned to its original value until well into the 1990s. By then, if they had lived that long, the couple would have been close to 100 years old, and for a number of years, their plans for a comfortable retirement might have deteriorated into poverty and need. Another scenario illustrates the risks of investing: A couple retires in the middle of 2008 with a big stock portfolio, planning to withdraw the minimum deduction (RMD) required by the Internal Revenue Service. Within a month, their portfolio would have been worth roughly one-half to two-thirds of its value when they retired. If they had panicked and sold their investments, they would have lost that much. On the other hand, if they had retained their portfolio for six years and cut back their lifestyle to still live on their RMD (perhaps supplemented by part-time work), their investment could have recovered almost fully. So investing can mean either/both making and losing

money. That is part of life. As teachers, it is up to us to help our students learn to calculate the risks so they can decide how to invest wisely.

In His parable of the talents, Christ showed that He understood the concept of investing money and earning interest on it.[19] Remember, the servant who received only one talent would have been commended if he had invested it.

Jesus, too, took risks. By coming to earth and engaging Satan on his own turf, He took the very real chance of eternal loss of connection with the Father. Yet He was willing to take that chance because He loves us that much.[20]

Statistics

Students and teachers are more or less familiar with concepts such as averages and standard deviations. They have also read news reports that include statistics relating to many kinds of phenomena. The U.S. Bureau of Labor Statistics publishes a huge set of statistics that is used to support all kinds of allocations of government money.

Statistics not only tell us about averages and deviations, they also give us a way to attempt to predict the future. Angelina Jolie is an Academy Award-winning American film star who was listed as Hollywood's highest-paid actress during several recent years.[21] She recently underwent a radical double mastectomy because of having been diagnosed with a mutation of the BRCA1 gene. She chose the surgery as a preventative measure because of the high probability that women with this gene mutation will get breast cancer.[22]

The Applications of Mathematics[23] text I used for my college class, "The Heart of Mathematics," uses an example about HIV/AIDS, an issue that should concern teen and young adult students in our worldwide church. It goes something like this:

Suppose you stop by your friend Alice's room one day, and you find her lying on her bed crying her heart out. She is sure she will die. She tells you that she just took an HIV test and tested positive. She says she was told that the test is 99 percent accurate (in other words, only one person in 100 who doesn't have the disease will test positive). In this particular test, 95 percent of those who are infected will test positive.

Naturally, you are in a far better position to think clearly than Alice is right now, so you set out to discover exactly what this test means for Alice. You remember that with a test like this one, there are four potential results, which can be placed in a table like this:

	Test Positive	Test Negative
Infected with HIV	95%	5%
Not infected	1%	99%

We'll use U.S. statistics for this computation; representative figures can be inserted for any area of the world. Through research, you establish the following facts:

a. There are roughly 320,000,000 people in the U.S.[24]

b. Of these, approximately 1,100,000 people in the U.S. are living with HIV.[25]

c. Of these people who have been diagnosed as HIV positive, 95 percent would test positive with this test. So there would be 1,045,000 who test positive.

d. Subtracting a – b, there are 318,900,000 people in the U.S. who aren't infected.

e. Of these 318,900,000 people, 1 percent or 3,189,000 will test positive.

f. Adding c + e, if everyone took this test, 4,234,000 would test positive for HIV.

These numbers could be put in our table like this:

	Test Positive	Test Negative
Infected with HIV	1,045,000	55,000
Not infected	3,189,000	315,711,000

Alice, then, is one of the 4,234,000 in the entire United States who would test positive if given this particular HIV test. If Alice actually has the virus, she would be one of the 1,045,000 who tested positive. So the probability that she actually has the virus is 1,045,000/4,234,000 which is less than 25 percent. So Alice certainly needs to be concerned (especially if she has engaged in risky behavior), but there is a greater chance that she is not infected than that she is.

Examples like this one can be analyzed with your students to help them understand what statistics actually mean. If students are presented with a problem like the ones faced by Alice or Angelina Jolie, we hope they will be able to make the rational decision and one that is the least damaging to themselves and others. A short class discussion on lifestyle choices and high-risk behavior when using an example like this may be very helpful to some students.

Teaching About the Misuse and Pitfalls of Statistics

• *Representative Samples.* You have undoubtedly heard statements like "Three out of four doctors recommend Hair-Beater[26] to restore the hair you've been losing!" What the advertisers hope you will think is that of the 893,851 doctors[27] in the United States, 626,079 of them would recommend Hair-Beater. Actually, what has probably happened is that the advertisers carefully picked a (small) sample of doctors, possibly only four, three of whom will recommend their product.

A classic example of using a sample that does not represent the whole population is the *Chicago Tribune*'s announcement that Thomas Dewey had won the 1948 U.S. presidential election. The victorious Harry Truman[28] couldn't resist gloating over the *Tribune*'s error.[29] It appears that the newspaper's embarrassing mistake occurred because their reporters relied heavily on pre-election polls and ended their exit polls too early.[30] Thus, they missed the late voters, the rank-and-file laborers who went to the polls at the close of the workday and who leaned more toward the Democratic Party. In other words, the sample was not representative of the total voting population.

• *Graphical Representations of Statistics.* The manner in which statistics are presented can lead to erroneous conclusions. For example, Terri Schiavo,[31] a Florida woman who suffered severe brain damage in 1990, was kept alive in a hospital with a feeding tube. Eight years later, when her husband requested that the tube be removed, Terri's parents vigorously disagreed. Eventually, a court ruled for the husband, and Federal courts upheld the ruling, so the feeding tube was removed, and Terri died in 2005.

After taking a poll to measure support for the courts' ruling, the Cable News Network (CNN) published graphics like those in Charts 1 and 2.[32]

Chart 1 appears to show clearly that Democrats,[33] by a large majority, put a lower value on the importance of extending human life by all means possible.[34] But the difference between the respondents from the different parties was not nearly as dramatic as it appeared. Note that the chart starts with 53 percent and goes up to only 63 percent. When people criticized CNN, it published a less misleading chart, using a vertical axis of 0 to 100 percent, which makes the differences between the results appear far less dramatic.[35]

Chart 2 depicted the results much more accurately: Americans from all three political persuasions supported the court's decision with about the same frequency. Note also that the margin of error meant that there were no statistically significant differences among the three groups.

Recently, some larger U.S. newspapers and various Websites have been doing fact checking on public officials' and political candidates' statements, advertisements, and press releases, which includes an analysis of their use of statistical data.[36]

Asking students to check the use of statistics and research supporting both sides of a contentious debate such as global warming or genetically modified organisms will give them insight into how various groups use (and abuse) statistics to try to convince the public that their position is the correct one.

In 1954, Darrell Huff published the book *How to Lie With Statistics,*[37] a delightful, easy-to-understand book that has stood the test of time. It is still available in print and also in electronic form. Huff's book illustrates various ways that statistics can be and are designed to mislead and sway public opinion. I encourage every teacher and every high school and college library to have a copy available.

Tessellations

Tessellations are patterns of geometric shapes that can be used to tile a floor (and go on forever in every direction) without overlapping or leaving gaps. The most common are repeating regular polygons: squares, triangles, or hexagons, similar to the pattern in Figure 1. Bees have been using this tessellation ever since they started making honeycombs.

A number of artists have incorporated tessellations in their illustrations and paintings,[38] and the designers of quilts often use a regular tessellation as the basis of their designs, like that of the Friendship Quilt in Figure 2.[39]

While the concept of tessellations is simple enough so that children of almost all ages can understand and enjoy it, the topic can be pursued more rigorously by studying the geometry of polyhedrons in geometry classes, which leads directly into a modern branch of

Chart 1.

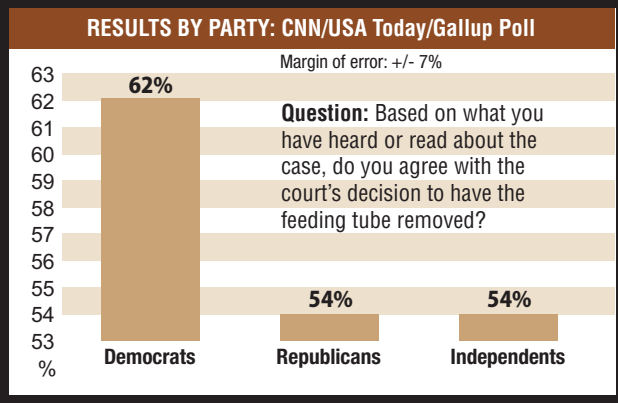


Chart 2.

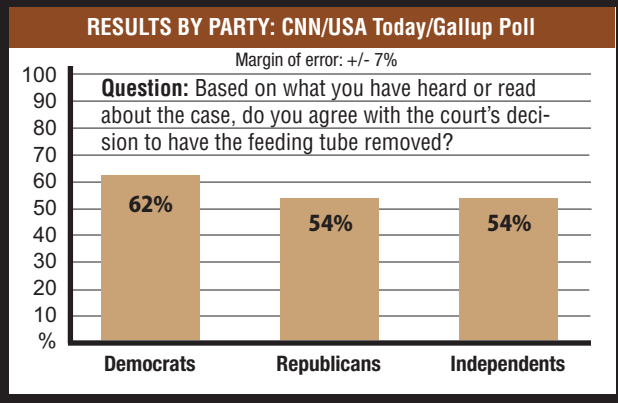
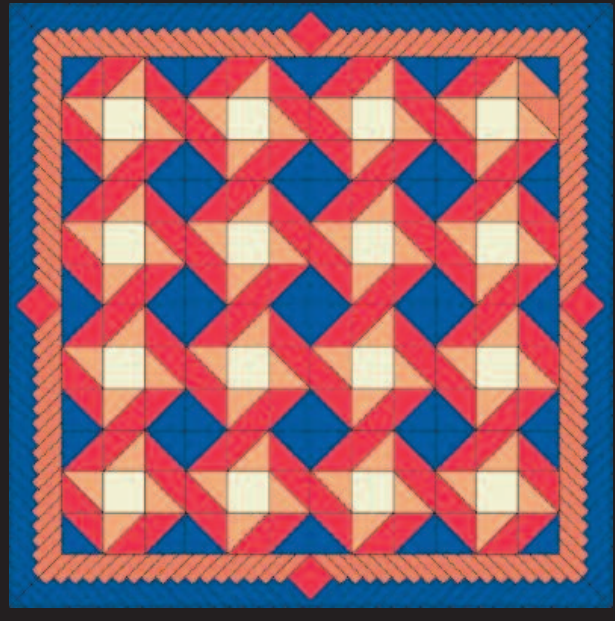


Figure 1.

Figure 2.



mathematics called transformational geometry.[40] From there, it is a natural step into linear algebra and matrix transformations.[41] For the practical minded, there is an extremely lucrative application of transformational geometry in designing video games.

Issues discussed in this article like avoiding debt, HIV/AIDS, interpreting statistics accurately, and understanding investing are important for improving human life and even possibly for continuing our existence. The last example, tessellations, is included to show that mathematics also provides the basis for fun and creativity. We, as math teachers, can help our students face life and the future with better confidence, avoid pitfalls like disease and debt—and just possibly see some reason for learning mathematics. ✐

This article has been peer reviewed.

Wil Clarke, Ph.D., *is retired in southern California. He taught mathematics and related subjects at Ikizu Secondary School in Tanzania, Helderberg College in South Africa, and in the United States, at Atlantic Union College, and most recently, at La Sierra University in Riverside, California. Dr. Clarke served as the Coordinator for the* JOURNAL'S *Math Issue in 2013.*

NOTES AND REFERENCES

1. The purpose of this article is to encourage teachers to include some practical applications in their classes that may well be useful to every student throughout his or her life. These include how to manage personal finance in a responsible way, interpret statistics, and recognize and enjoy numerical, geometrical, and other patterns that they will encounter. I drew some of the examples in the article from those I have used in my Applications of Mathematics class, which includes topics from a variety of areas. For additional information on topics that might be covered in a practical mathematics article, contact the author at wil.clarke@gmail.com.

2. http://www.adventist.org/fileadmin/adventist.org/files/articles/officialstatements/28Beliefs-English.pdf.

3. http://money.cnn.com/2013/05/17/pf/college/student-debt/.

4. http://studentaid.ed.gov/repay-loans/understand/plans.

5. http://studentaid.ed.gov/types/loans/interest-rates#what-are-the-interest-rates-of-federal-student-loans.

6. I supplied my Applications of Mathematics students with the following three formulas:

Compound Interest: $$Amount = principal\left(1+\frac{rate}{number}\right)^{number\ x\ years}$$

Monthly Investment: $$Payment = amount\ x\left(\frac{rate/12}{(1+rate/12)^{months}-1}\right)$$

Monthly Payment on Loan: $$Payment = \frac{principal\ x\left(1+\frac{rate}{12}\right)^{12\ x\ years}\ x\left(\frac{rate}{12}\right)}{\left(1+\frac{rate}{12}\right)^{12\ x\ years}-1}$$

7. http://money.cnn.com/magazines/moneymag/money101/lesson9/index2.htm.

8. Many thanks to one of the peer reviewers for suggesting this site.

9. http://www.nsc.org/news_resources/injury_and_death_statistics/Documents/2014-Injury-Facts-43.pdf.

10. For example, see http://www.science20.com/gerhard_adam/flying_or_driving_which_safer.

11. https://www.facebook.com/notes/quicken-loans/quicken-loans-billion-dollar-bracket-challenge-short-form-rules/10152185833560489.

12. http://en.wikipedia.org/wiki/Lightning_strike.

13. See http://www.nlm.nih.gov/medlineplus/ency/article/001520.htm, which quotes the National Institutes of Health's definition of pathological gambling.

14. http://www.adventist.org/information/official-statements/statements/article/go/0/gambling/.

15. See http://www.scotland.gov.uk/Publications/2006/08/17134534/6 paragraph 5.89; and http://www.washingtonpost.com/wp-srv/local/longterm/library/lottery/lottery1.htm.

16. See, for example, pages 39 and 129 of http://www.adventist.org/fileadmin/adventist.org/files/articles/official-statements/Statements-2010-english.pdf; and the article by Arthur F. Blinci and Tim Northrop in the October/November 2014 issue of the *Journal* titled, "Insurance 101—Is Your Campus Properly Insured?" on pages 40 to 47.

17. See information on the Seventh-day Adventist Retirement Plan for the North American Division at http://www.adventistretirement.org/site/1/docs/SDA%20SPD%202012.pdf.

18. http://www.macrotrends.net/1319/dow-jones-100-year-historical-chart.

19. Matthew 25:14-28.

20. John 3:16.

21. http://en.wikipedia.org/wiki/Angelina_Jolie.

22. http://www.cnn.com/2013/05/14/showbiz/angelina-jolie-double-mastectomy/.

23. This discussion is an adaptation from the third edition of *The Heart of Mathematics* by Edward B. Berger and Michael Starbird (Hoboken, N.J.: J. Wiley, 2010), pages 813-816.

24. http://www.census.gov/popclock/. This site keeps a running estimate of the number of people in the U.S. and also in the world.

25. http://www.cdc.gov/nchhstp/newsroom/docs/Vital-Signs-Fact-Sheet.pdf.

26. This is a name I created for illustrative purposes and does not refer to any existing product.

27. http://kff.org/other/state-indicator/total-active-physicians/.

28. http://www.canyoncountryzephyr.com/blog/wp-content/uploads/2012/10/truman.jpg. Other leading publications and prominent broadcasters had also predicted Dewey's victory: http://www.eagleton.rutgers.edu/research/americanhistory/ap_trumandewey.php.

29. http://www.chicagotribune.com/news/nationworld/politics/chi-chicagodays-deweydefeats-storystory.html.

30. http://www.eagleton.rutgers.edu/research/americanhistory/ap_trumandewey.php.

31. http://en.wikipedia.org/wiki/Terri_Schiavo_case.

32. http://www.relentlesslyoptimistic.com/2005/03/statistics_and_.html.

33. In the U.S., the Democratic Party is often alleged to value human life more lightly than other parties. CNN may have been, intentionally or unintentionally, trying to take advantage of this perception.

34. http://mschindler.com/2005/03/22/partisan-shmartisan/.

35. http://www.democraticunderground.com/top10/05/191_cnn2.gif.

36. http://www.technorms.com/454/get-your-facts-right-6-fact-checking-websites-that-help-you-know-the-truth.

37. *How to Lie With Statistics* by Darrell Huff, reissued by W. W. Norton in 1993.

38. For example, M. C. Escher: http://www.mcescher.com/.

39. http://tlc.howstuffworks.com/home/friendship-quilt-pattern.htm.

40. David A. Thomas, *Modern Geometry* (Boston: Cengage Learning, 2001).

41. David Poole, *Linear Algebra: A Modern Introduction,* 3rd edition (Boston: Brooks/Cole, 2011).