Response to Intervention for Math

Using Data to Design and Monitor Interventions

In 2013, only 42 percent of 4th-grade students and 34 percent of 8th-grade students in the United States scored at or above the proficiency standard on the National Assessment of Educational Progress in math,[1] and approximately 30 percent of the 4th-grade students and 20 percent of 8th-grade students who completed the Trends in International Mathematics and Science Study[2] demonstrated proficient math skills.

Math proficiency requires both procedural fluency and conceptual understanding.[3] *Procedural fluency* is the understanding of the rules and steps that make it possible to solve a math problem as well as the ability to use them quickly enough to compute results; while *conceptual understanding* is the knowledge of the underlying relationships that make the procedure work.[4] Researchers have found that many students have adequate conceptual understanding but struggle to recall basic math facts quickly enough to apply the information within a procedure.[5] Furthermore, remedial instruction that focused on math fact fluency improved students' recall of math facts, computational fluency, and performance on different types of math problems,[6] but had less of an effect for students who lacked a basic understanding of the underlying concepts.[7] Thus, the procedural and conceptual knowledge dichotomy could be useful for determining what type of math intervention specific students need.

What Is Response to Intervention (RTI)?

Response to Intervention (RTI) is a systematic process to analyze student outcome data in order to determine appropriate interventions for students before considering whether they have a disability. Once it is determined that a learning disability exists, special-education professionals use RTI outcome data to continue providing intensive instructional support to students who need it. As a result, RTI approaches work well for students who lack academic preparation and are at risk, as well as those who are diagnosed with learning disabilities.

Several practices are associated with RTI, including providing high-quality instruction, changing instruction by frequently monitoring student progress, and making important

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educational decisions based on student response to the changed instruction or intervention.[8] The main components of RTI are (1) quality core instruction, (2) universal screening, (3) monitoring progress of students identified with difficulties, (4) implementation of increasingly intensive interventions based on student need, and (5) use of resulting data collected to make instructional and resource allocation decisions.

As shown in Table 1 (and discussed elsewhere in detail in Burns and Gibbons, and VanDerHeyden and Burns[9]), RTI is most effectively delivered within a three-tiered model of intervention. Tier 1 consists of quality core instruction and universal screening three times each year; Tier 2 generally involves small-group interventions that target broad academic deficits; while Tier 3 interventions are usually intensive approaches designed for and delivered to individual students. Thus, as student needs and difficulties become more intense, the required interventions will need to become more intense (i.e., require more resources and involve more analysis to identify them).

Essentially, an RTI system revolves around attempting to answer the three analysis questions presented in Table 1. Thus, measurement becomes more frequent and more precise within each tier.[10] Assessments for Tier 1 decisions are conducted three times each year but may be required daily or at least once or twice each week for Tier 3. Second, progressing through the tiers, measurement becomes more precise. For Tier 1, general outcome measures (GOMs) involve gross measures of overall math proficiency. However, decisions at Tiers 2 and 3 are based on data from much more specific subskill measures such as mastery of specific math facts or concepts (e.g., single-digit multiplication; division of double-digit numbers by a single-digit number without remainders; multi-digit addition and subtraction; time, money, and word problems). Both of these two progressions (frequency and precision) occur because the specificity of problem solving increases as students progress through the tiers. Essentially, an effective RTI model attempts to answer three questions:

- The primary problem analysis for Tier 1: Is the problem specific to the student or the system? (Is it a class-wide problem?)
- The Tier 2 problem analysis question: What is the category of the problem?
- And finally, Tier 3 attempts to determine the following: What is the causal variable?

The term "causal variable" can be confusing. As used here, the term refers to the environmental variable (e.g., amount of

repetition, focus of the intervention, and approximation to how information will be applied) teachers can manipulate and that is most closely related to the problem. To illustrate, let's use this framework to discuss an RTI model for math.

Tier 1 – Class-wide Problem

The definition of quality mathematics instruction has generated considerable debate, and describing the characteristics of core instruction are beyond the scope of this article. Research has highlighted the need to begin RTI decision making by first identifying class-wide problems as an indicator of the quality of core instruction.[11] If a class-wide problem exists, then it may be more efficient to conduct the intervention for the entire class than to pull students out and perform interventions at the selected level (Tier 2). For example, consider a 3rd-grade classroom with 25 students, 14 of whom score below the expected level of performance on standardized tests or other forms of assessment and thus require intervention. It would be impractical to provide small-group intervention for 14 of 25 students. Thus, data from the assessment should be used to identify class-wide problems and compare data.

Data

Data used for Tier 1 decisions within an RTI framework should be global in nature or associated with an entire curriculum because their low-level analyses only examine students' overall proficiency. Thus, most measures used within Tier 1 are considered general outcome measures that assess proficiency of global outcomes.[12] Below, I will discuss two common approaches for performing Tier 1 assessments.

Curriculum-based measurement. Many schools use curriculum-based measurement (CBM) to screen for math performance because it is easy to use, free to create, and has strong evidence for reliability and validity.[13] When using the CBM approach, teachers provide the students with a series of math computation problems, provide four minutes for them to complete the problems, then record the number of correct responses per minute. Thus, data from a math CBM consist of digits correct per minute (DCPM). (To explain the DCPM metric would exceed the scope of this article. Readers are encouraged to consult Hosp, Hosp, and Howell's book on curriculum-based measurement[14] or the Websites in Table 2.)

Math CBM either samples a single skill (e.g., single-digit

Table 1. Problem Analyses and Data Within Three Response to Intervention Tiers

Tier	Primary Problem Analysis Question	Data Used	Intervention Focus
Tier 1	Is there a class-wide problem?	General outcome measure (GOM) (e.g., curriculum-based measurement or group-administered standardized math test) three times each year	Class-wide interventions for procedural fluency
Tier 2	What is the category of the problem?	Designing Interventions: • GOM to screen • Subskill mastery measures (SMM) (e.g., curriculum-based assessment for instructional design [CBA-ID] of specific objectives) Monitoring Progress: • Weekly GOM and SMM	Procedural fluency of specific math objectives
Tier 3	What is the causal variable?	Designing Interventions: • GOM to screen • SMM of specific objectives • Conceptual understanding assessment Monitoring Progress: • At least weekly GOM and SMM	Intense procedural fluency or conceptual understanding of specific objectives

Table 2. Website Resources for Response to Intervention (RTI)

Area	Group	Address
RTI Overview	RTI International	http://www.rti.org/
	National RTI Center	http://www.rti4success.org/
Math curriculum-based measurement	Vanderbilt University	https://my.vanderbilt.edu/specialeducationinduction/files/2013/07/IA.Math-CBM.pdf
	National Center on Student Progress Monitoring	http://www.studentprogress.org/
	Intervention Central	http://www.interventioncentral.org/curriculum-based-measurements-reading-math-assesment-tests
Interventions	Evidence-based Intervention Network iSTEEP	http://ebi.missouri.edu/ http://gosbr.net/

multiplication, three-digit subtraction with regrouping, etc.) or involves a multi-skill probe (i.e., combination of problems from the grade-level curriculum). A multi-skill probe allows practitioners to craft an assessment that samples all of the yearly objectives for a given grade, which may more effectively highlight students' progress over time. Moreover, multi-skilled measures correlate better with outcome measures of math performance than do single-skill probes.[15] Thus, CBM multi-skilled probes can be often used as efficient indicators of general math skills within Tier 1 and thus are administered three times each year as a universal screening tool. Classroom teachers will require much more detailed data than these to drive daily instruction, but math CBM data can adequately screen students to determine which ones need additional support and to identify a class-wide problem.

Group tests. Although CBM math assessments can produce useful screening data, many teachers are also interested in assessments of math concepts and applications, which can also be reliably measured.[16] Thus, some schools conduct universal screenings three times each year using highly reliable group-administered commercial measures such as the Measures of Academic Progress[17] or Star Math.[18] With commercially prepared math screeners, the data tend to link better to instruction than CBM math,[19] and Star Math has predicted state test scores among 3rd- and 4th-grade students better than did math CBM data.[20] Moreover, both Measures of Academic Progress and Star Math are computer-adaptive tests, which ensure efficient measurement with psychometrically sound data because groups of students can complete the tests in a matter of 15 to 30 minutes, and the tests can be instantaneously scored and reported.

Readers in countries other than the United States are encouraged to examine measures created with the EdData II project.[21] Some measures are available free on the Website, and most have convincing evidence for reliability and validity.

Criteria

To interpret screening data, the teacher must first identify or rule out class-wide problems. To do this, he or she must examine the score for every student and calculate the class median. The median is used because averages can be disproportionally affected by outlying data for small data sets (e.g., less than 30 data points), and this measure is not susceptible to skewing by outliers. The class median is then interpreted by comparing the score to a national standard. Class medians that fall at or below the 25th percentile on the national norm (the lowest end of the average range) suggest a class-wide problem.[22] However, the criterion used to identify a class-wide problem is a local policy decision. Choosing the 50th percentile as the cutoff point for intervention obviously represents a more ambitious standard that may be more appropriate for some schools. Criteria for various decisions are included in Table 3. Scores presented for screening decisions represent the 25th percentile on a national norm for multi-skilled CBM for math.

Intervention

If the teacher identifies a class-wide problem, it may be more efficient to bring the intervention to the class than to pull students out for intervention at the selected level (Tier 2). VanDerHeyden and Burns[23] implemented a peer-assisted learning strategy to teach math facts in schools with a class-wide problem (see http://www.gosbr.net/math/ for a protocol). The classroom intervention led to such a dramatic improvement in student skills that the class-wide problem was no longer evident after a few weeks, and student scores increased significantly within the school year and across cohorts.[24]

Class-wide interventions generally require approximately 10 to 15 minutes per day. Students form heterogeneous pairs, practice single-skill computation for two minutes with worksheets or flashcards, after which they score each other's work and provide feedback on its accuracy.[25] Students are then assessed each week, and the intervention continues until the class median exceeds the criterion.

VanDerHeyden and colleagues[26] have consistently demonstrated the effectiveness of identifying and remediating class-wide problems in math. Intervention protocols that utilize peer tutoring to deliver fact-fluency interventions have been shown to be effective in improving student achievement.[27]

Table 3. Criteria for Math Assessments in Digits Correct per Minute

Grade	Instructional Level[1]	Screening[2]	Proficiency[3]	Rate of Growth[4]
		2 Fall		.40 per week Realistic
First	14 to 31	7 Winter	17	.65 per week Ambitious
		10 Spring		.30 per week 50th percentile
		8 Fall		.30 per week Realistic
Second	14 to 31	16 Winter	17	.50 per week Ambitious
		16 Spring		.30 per week 50th percentile
		12 Fall		.20 per week Realistic
Third	14 to 31	18 Winter	17	.35 per week Ambitious
		21 Spring		.40 per week 50th percentile
		24 Fall		.45 per week Realistic
Fourth	24 to 49	32 Winter	29	.75 per week Ambitious
		39 Spring		.50 per week 50th percentile
		23 Fall		.30 per week Realistic
Fifth	24 to 49	28 Winter	29	.50 per week Ambitious
		36 Spring		.50 per week 50th percentile

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Tier 2 – Category of the Problem

The second question within an RTI framework involves identifying the category of the problem. Because math is extremely objectives-based, objectives can be used as categories to identify intervention targets. For example, a student may be struggling with single-digit multiplication, or solving word problems, or addition of fractions with like denominators. Practitioners can assess student skills in specific math objectives to determine the appropriate starting point for intervention. Once the target is identified, students can be grouped (e.g., students who require intervention in single-skill multiplication in one group, and students who struggle with triple-digit subtraction with regrouping in a different group, etc.). The data, criteria, and interventions to use for this process are described below.

Data

Data used for Tier 2 decisions are more specific than mere measures of general math proficiency. The following section provides information about how to use curriculum-based assessment for instructional design and technology-enhanced approaches to assess individual skills.

Curriculum-based assessment for instructional design. The level of problem analysis increases in complexity as children progress through the three tiers, which indicates that decisions made in Tiers 2 and 3 depend on more specific subskill mastery measures (SMMS)[28] such as curriculum-based assessment for instructional design (CBA-ID).[29] CBA-ID measures only one objective at a time (e.g., single-skill multiplication), but the assessment is conducted similarly for CBM. Students receive a sheet with computation problems representing one skill (e.g., single-digit multiplication) and have two to four minutes to complete as many problems as possible. Once again, the data are obtained by computing digits correct per minute. However, in Tier 2, the data are compared to a criterion for an instructional level, which will be discussed below. Thus, CBA-ID is not identical to CBM, even though they have much in common because the data are interpreted somewhat differently and are based on only one type of skill.

Although constructing CBA-ID probes is largely a straightforward process, skill sequencing is important. Many free Websites offer help in creating single-skill probes including www.mathfactcafe.com, www.aplusmath.com, and www.interventioncentral.com. After creating the probes, the sequence of the skills within the curriculum will determine the order of the probes. Since many curricula do not have a research-based skills sequence, practitioners will need to rely on the sequence in which skills are actually taught.

Technology-enhanced assessment. Although formative evaluation is a critical component of effective instruction and intervention, a truly formative and comprehensive assessment-to-intervention system is difficult to implement.[30] Technology-enhanced assessment can make continuous assessment more manageable[31] and help ensure more systematic diagnostic assessment and monitoring of student progress.[32]

Accelerated Math[33] and Monitoring Basic Skills Progress (MBSP)[34] are two potentially useful technology-enhanced math assessments. Accelerated Math assesses specific objectives, provides targeted practice of deficit skills until mastery, and includes regular review of mastered objectives for skill maintenance. Accelerated Math also provides an automated feedback loop, and the data generated can be used to target interventions. The MBSP is a somewhat similar approach that includes measures of Basic Math Computation, and Basic Math Concepts and Applications. The MBSP involves students completing assessments that are instantaneously scored and included in graphs of student progress, class-wide reports, and instructional recommendations.

Criteria

Data are used for two purposes within Tier 2: (1) to determine which intervention to use; and (2) to determine if the intervention is working. The criteria for selecting interventions and monitoring progress are discussed below.

Selecting interventions. CBA-ID data are usually interpreted with instructional-level criteria. Burns, VanDerHeyden, and Jiban[35] empirically derived math criteria to which SMMs could be compared. The results of this study found instructional-level criteria of 14 to 31 DCPM for 2nd- and 3rd-graders, and 24 to 49 DCPM for 4th- and 5th-graders, which are included in Table 3. Scores below the lowest end of the instructional level range fall within the frustration level, suggesting that the skill is too difficult for the child, while those that exceed the highest score of the instructional level range fall within the mastery (or independent) category. Thus, practitioners should administer single-skill probes until the task that represents an instructional level is identified for students in Tier 2 using the criteria derived by Burns and colleagues[36] for students through the 5th grade.

Monitor progress. Frequent monitoring of student progress is a hallmark of an effective RTI model. The more frequently data are collected, the more quickly decisions can be made. First, practitioners should evaluate the skill level. Although the 25th percentile can serve as an appropriate score to identify a student as struggling in math, it is too low a criterion to use as an indicator of proficiency. Some practitioners may use the 50th percentile, which may be appropriate. Likewise, the instructional-level criteria derived by Burns and colleagues[37] may be useful for designing interventions, but higher criteria may be needed to determine proficiency. VanDerHeyden and Burns[38] empirically determined the score necessary at various grade levels to determine proficiency, i.e., the one that will lead to mastery of subsequent skills and may be appropriate targets for proficiency with CBA-ID data.

After determining if the student has reached a level of proficiency, the teacher should assess his or her rate of growth. There are four possible decisions: (1) the student has reached proficiency, and intervention is no longer needed; (2) the student is making adequate progress but has not yet achieved proficiency, and the intervention should continue; (3) the student is not making adequate progress, has not yet reached proficiency, and a change is needed within the tier; and (4) the student is not making adequate growth, has not reached proficiency, and a change in tier is needed (e.g., from Tier 2 to Tier 3). As can be seen from the list of possible decisions, rate of growth is an important indicator within an RTI framework.

Using both level and growth to judge student progress results in better decisions than using just one of these measures.[39] Multiple methods can be employed to examine growth including normative approaches such as using the 50th percentile rank in rate of improvement on a national norming.[40] The rate of growth (in increased digits correct per minute per week of intervention) that represents the 50th percentile for each grade is included in Table 3. Fuchs, et. al.,[41] presented math growth rates for realistic and ambitious goals that may also be useful, to judge the growth of individual students, and which are also included in Table 3.

Intervention

Fewer commercially prepared small-group interventions exist for math than for reading, but research has identified potential targets and approaches. Math interventions should include explicit instruction that involves models for solving a problem type using an array of examples, extensive practice in use of newly learned strategies and skills, opportunities to think aloud, and extensive feedback.[42] Fluent computation is also an important goal for math and could be a target for small-group interventions since students who struggle in math often have trouble recalling basic facts[43] and lack the skills required to solve more advanced problems.[44] Moreover, providing additional practice with basic or component skills (e.g., single-digit multiplication) led to increased performance of the more advanced skills.[45]

Several commercially prepared interventions can be used to enhance students' fluency in basic math skills while also providing an array of problem examples, extensive practice in use of newly learned skills, and in-depth feedback. Technology-based drills, practice, and tutorials can improve student performance in specific areas, but research relating to this has been inconsistent.[46] Math Facts in a Flash (MFF)[47] is a computer-delivered math intervention that improved classroom-level math computation skills[48] among students receiving a Tier 2 intervention for math.[49] A relatively large number of students can participate in MFF simultaneously because it is delivered via computer. Thus, MFF offers a potential small-group Tier 2 intervention for math. However, previous research used other approaches to help students practice math fluency while providing an array of problem examples and extensive feedback such as flashcards with student dyads and paper-and-pencil assessment.[50] Several easy-to-use group math-intervention protocols developed from research by Amanda VanDerHeyden are available free of charge at www.gosbr.net/math.

Tier 3 – Causal Variable

The primary problem-analysis question for Tier 3 addresses the causal variable; that is, the variable teachers can control which is most closely related to the problem. Extensive data analysis exists for Tier 3 decisions, which will be discussed below.

Data

As with Tier 2, data for Tier 3 are needed to monitor student progress and to analyze student difficulties. The framework for

monitoring progress at Tier 3 is essentially identical to Tier 2 but is conducted with greater frequency. Teachers monitor student progress at least once each week, assessing both subskill mastery and general outcome measures. Students receiving a Tier 2 intervention are assessed using a general outcome measure (e.g., multi-skilled CBM Math, Star Math) every other week, but those receiving a Tier 3 intervention need to be monitored weekly or even more frequently. This ensures that instructional changes can be made promptly, as contrasted with less-frequent assessment.

Children receiving Tier 3 interventions receive much more complex problem analysis than for those at Tier 2 because general measures alone provide insufficient data for instructional planning. Rivera and Bryant[51] present both active and passive means to assess the procedures students used to complete the task.

The passive approach examines error patterns by examining samples of completed tasks. For example, a child may complete a subtraction problem with regrouping by simply subtracting the smaller number from the larger regardless of where it is placed in the problem.

Passive approaches are often sufficient for classroom teachers to identify common errors that suggest which potential skills or procedures need to be retaught, but a more in-depth approach will probably be necessary for a student receiving a Tier 3 intervention. Thus, teachers may conduct one of two active assessments to target intervention efforts: (1) Provide the student an example of the target problem and ask him or her to "think out loud" while completing it. This will allow the student to verbally articulate what errors are being made[52]; (2) If the error is not articulated, then the student should be provided with a second example of the problem type, asked to assume the role of teacher, and explain to the instructor how to complete the task. One of these approaches will likely identify the specific procedural error being made, which can then be directly and explicitly remediated.

Although procedural information is important for analyzing math difficulties at Tier 3, creating interventions for children with more severe difficulties will likely require additional strategies beyond mere procedural indicators. Simple discrimination tasks can be used to assess students' conceptual understanding of mathematical concepts by asking them to judge whether items are correctly completed.[53] For example, a student could be provided a series of items, each of which contains three examples of the same mathematical equation, and asked to circle the correct one. Building on research by Beatty and Moss,[54] Burns[55] developed a math measure containing two equations for each item and a picture cue (e.g., two dice) that matched one of the equations. Students were instructed to circle which of the two equations matched the picture. The data were used to determine whether the students understood the concept of single-digit multiplication, and the appropriate intervention (i.e., conceptual versus procedural) produced positive effects.

Criteria

Data used to monitor student progress at Tier 3 should be compared to the same criteria used in Tiers 1 and 2. For example, general outcome measures should be compared to research-based benchmarks for proficiency,[56] and subskill mastery data should be compared to empirically derived criteria for an instructional level.[57]

Intervention

Although there are fewer interventions for math than for reading, meta-analytic research has identified important components of an effective intervention. First, intensive interventions within Tier 3 should use principles of explicit instruction in teaching both math concepts and procedures.[58]

However, the same study found a negligible effect for conceptual math interventions and reported that these interventions "present a complex puzzle of findings, open to multiple interpretations."[59] One explanation for the complex findings relating to conceptual interventions could be that many students who struggle with math understand the underlying concept but require additional intervention for the procedure,[60] and differentiating between the two provides a useful intervention heuristic.[61]

Effective computation interventions include practice with modeling and various strategies for providing high repetition with the task.[62] For example, Incremental Rehearsal (IR)[63] has consistently proved effective in providing high repetition of basic math facts.[64] Readers are referred to http://www.interventioncentral.org/academic-interventions/math-facts/math-computation-promote-mastery-math-facts-through-incremental-re for additional information on using IR with math facts. However, there are many good sources for conceptual interventions as well.[65]

Conclusion

Although RTI procedures for reading remediation are well articulated, math research has been considerably less prominent over the past decade.[66] Some aspects of the RTI framework cut across reading and math, such as first examining class-wide problems, delivering quality core instruction, screening all students, closely monitoring student progress, and implementing increasingly intensive interventions based on student need. However, different assessment data are needed for reading than for math, and the problem analysis approach at Tiers 2 and 3 could be qualitatively different (e.g., category of the problem based on objectives and assessing conceptual understanding within Tier 3 decisions). The approaches to intervention suggested in this article provide schools with a place to begin implementing Response to Intervention; however, this area continues to be one that requires more research as schools pay close attention to math instruction and math outcomes. ✐

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